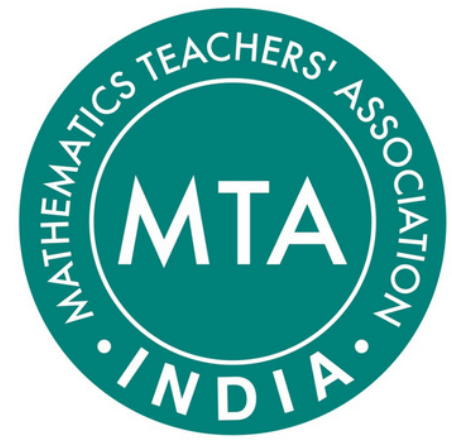




Euclid’s Elements in India

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Abstract

Elements is a pioneering mathematical treatise on geometry and number theory authored by Euclid around 300 BCE in Alexandria. It was written in Greek and consists of 400 propositions spanning 13 books. Its influence spread across the world with translations into various languages.⁸ The book reached India in the 18th and 19th centuries and has been translated into several Indian languages. These translations remain inaccessible to the students and people interested in geometry. As a resource, the books can be useful in the following aspects: (1) ensuring accessibility of an important text to native language learners (2) allowing students to sharpen geometry reasoning in their native language (3) providing an opportunity to understand the historical context of geometry.

About *Elements*

- Some people consider *Elements* to be next only to the Bible in its influence on the world.¹¹
- Its influence is not limited only to Mathematics; it has influenced art, philosophy, theism, printing techniques etc.⁶
- For example, Abraham Lincoln used a common notion from *Elements* to argue for abolition of slavery in the United States.⁶
- Modifying one of the postulates in *Elements* led to the development of other geometries, resulting in paths to new theories of the universe such as the Theory of Relativity.

Structure of the text

- Only a compass and straight edge were used.
- Measurements were not used.
- Propositions are interrelated.
- Propositions are stated and proved one following the other in a structured manner.

Definitions: Defining what lines, points, angles and their types, circles, radius, diameter and so on are.

Postulates (Post.) and Common Notions (CN):

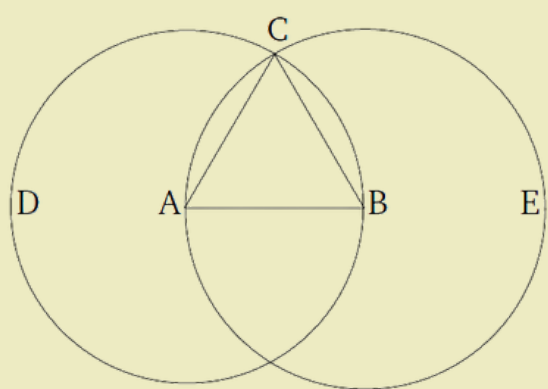
Statements that are accepted and have not been proved.

Propositions (Prop.): Results which have to be proved.

The Method of Proof:

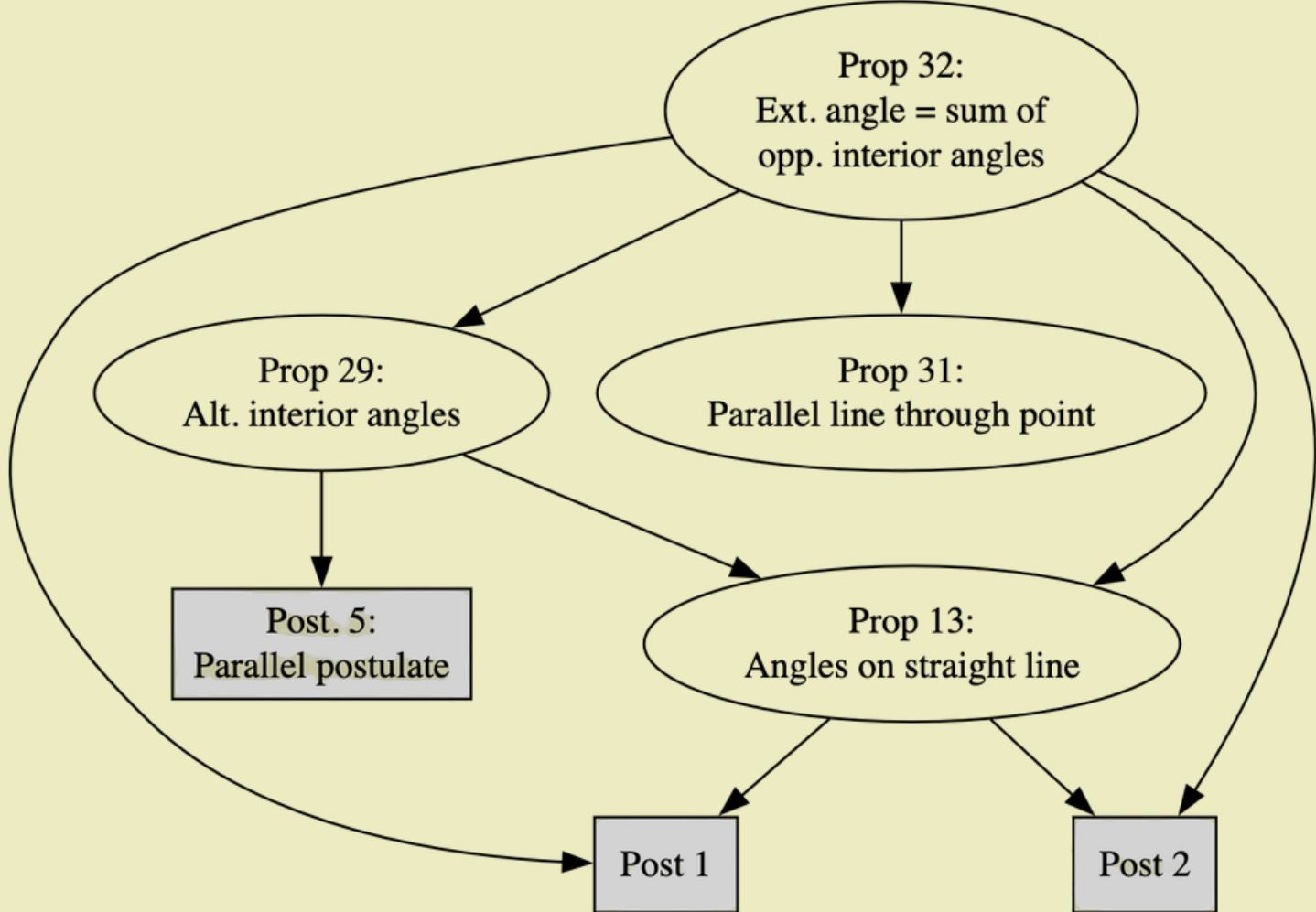
- Make a statement
- Draw a diagram to represent the problem with points labelled using letters
- Write out a chain of reasoning (using Post., CN and previous Prop.)
- conclude that the proposition is proved.⁸

Examples from *Elements*



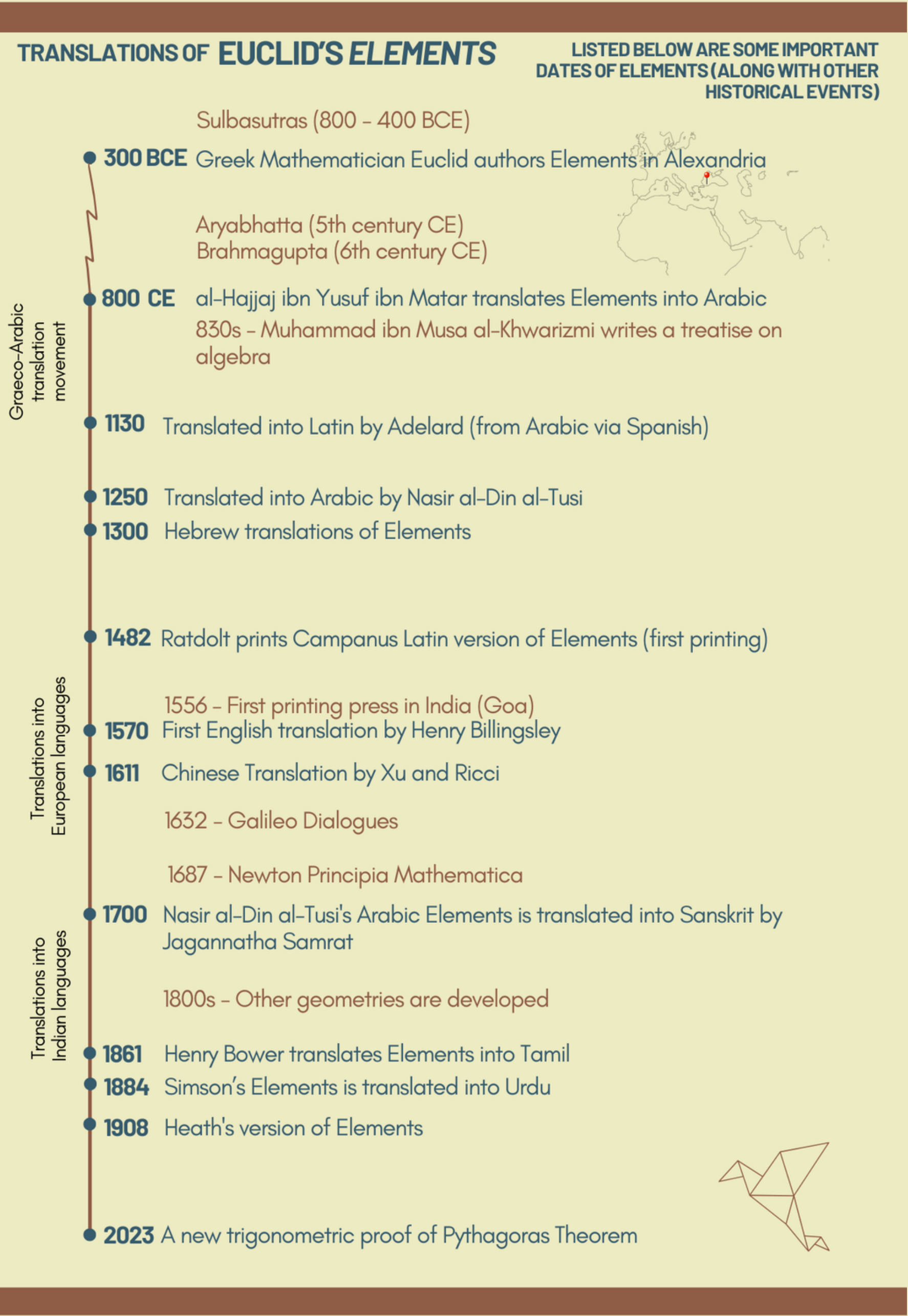
- AB is given.
- Draw circle with A as center and AB as radius, and another with B as center and BA as radius.

Constructing an Equilateral triangle (Prop. 1) during the time of Euclid



Proposition 32 (External angle - Internal angles relationship in a triangle and sum of angles in a triangle) dependencies in Book 1 of *Elements*

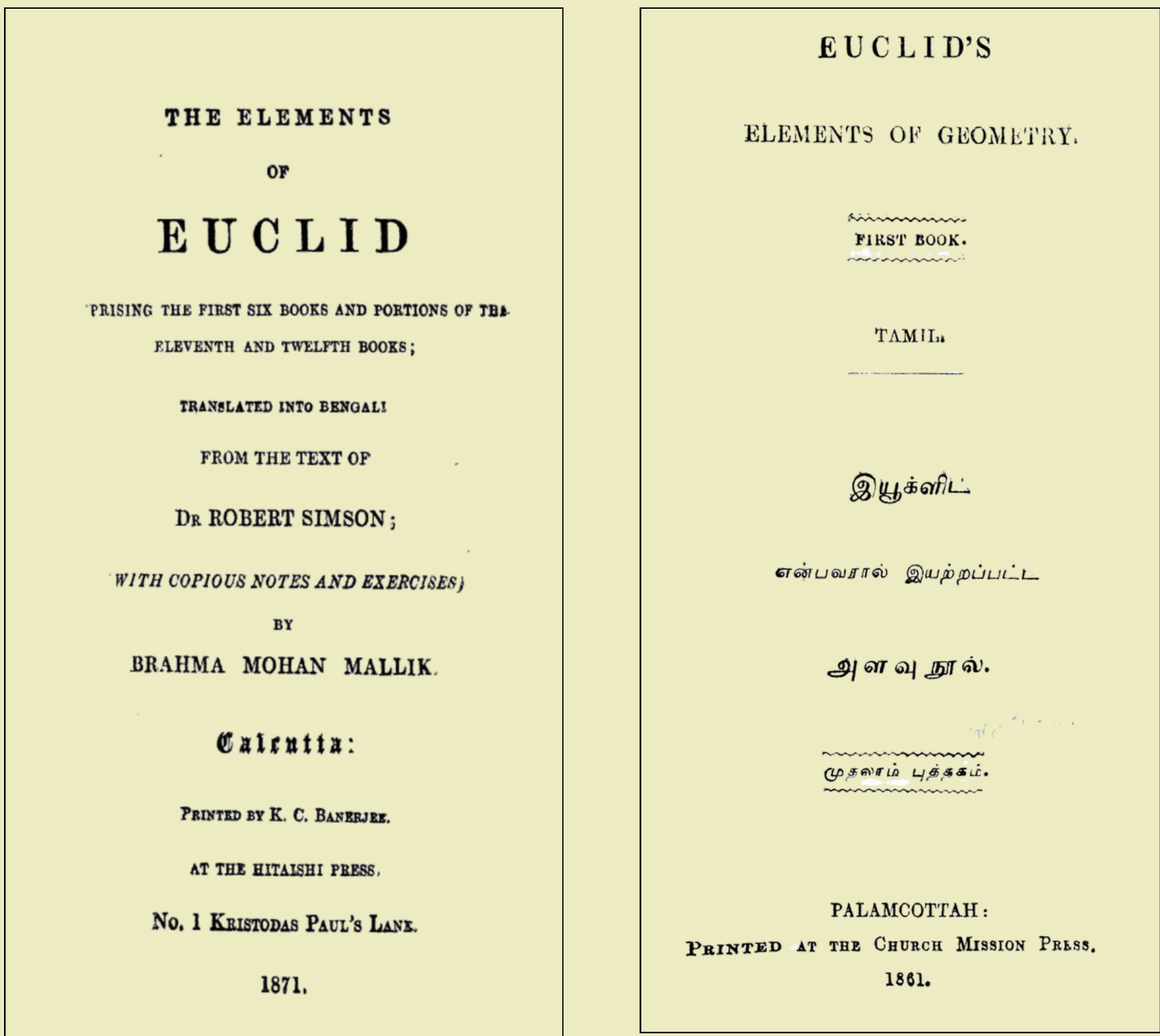
Elements around the world



A timeline showing the various translations of *Elements* alongside some historical events/works

Elements in India

- *Elements* was brought to India before British rule. Jagannatha Samrat had translated it into Sanskrit. He coined new words in Sanskrit while doing the translation.⁷
- In 1800s it was translated into many other Indian languages such as Sanskrit, Marathi, Bengali, Hindi, Oriya, Tamil^{1, 3, 4} Some of these are available on the Internet.

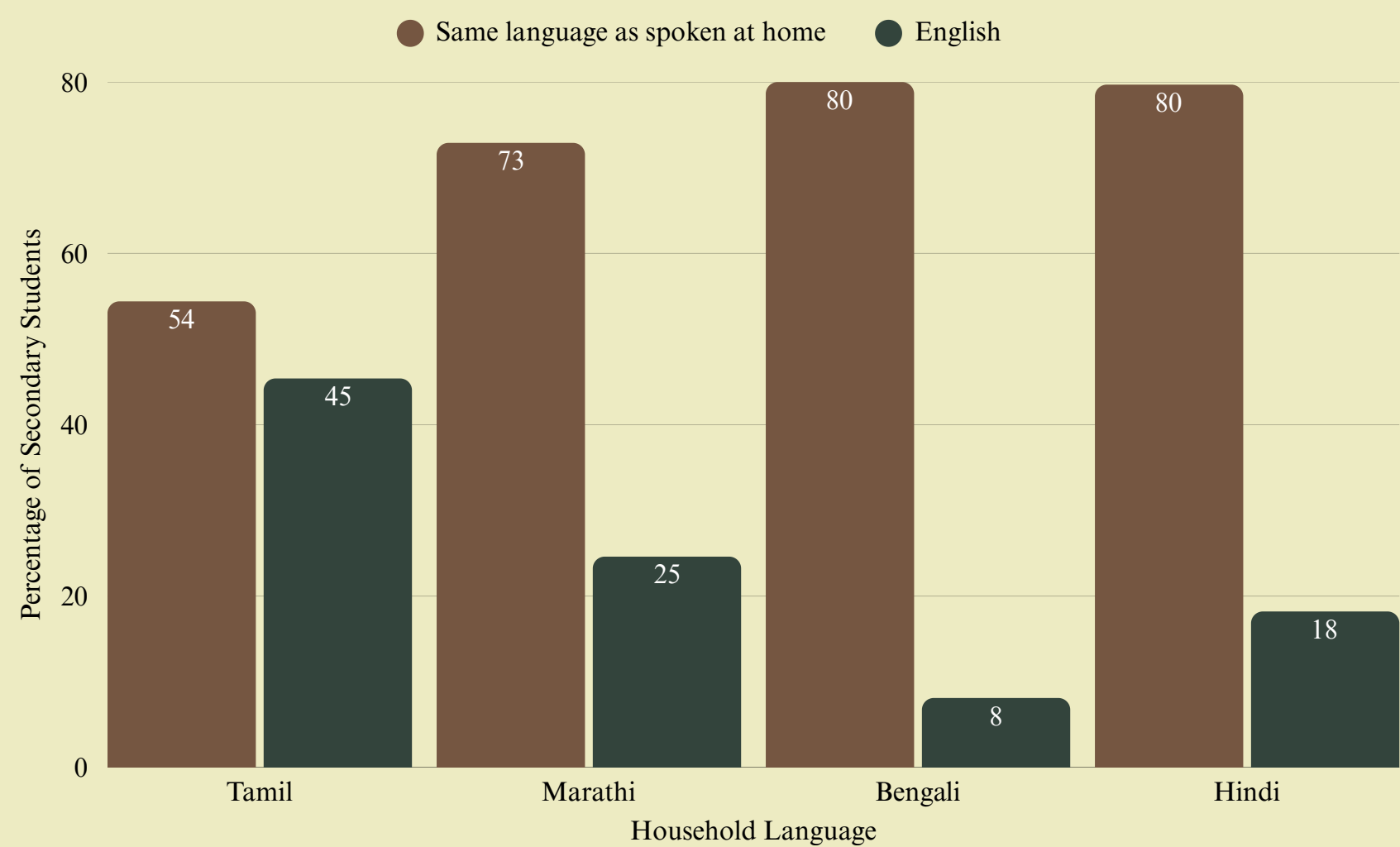


Bengali³ (1871) and Tamil⁴ (1861) translations of *Elements*

- There is a possibility that many translations of *Elements* in various Indian languages might exist.

Why *Elements* can be important as a resource book?

- The availability of the text in a student’s native language can help them sharpen geometry reasoning.²



(2017-2018) Medium of Instruction of Secondary School Students - NSS Report No. 585: Household Social Consumption on Education in India⁵

- An axiomatic approach to geometry is built from scratch in *Elements*. This structured approach to geometry in the *Elements* is markedly different from that used in textbooks, which can give the student broader perspectives.
- The text shows how geometry was done in the past and how it has evolved. In *Elements*, numbers were not used to measure geometric objects. For example: In Pythagorean Theorem (Prop. 47, Book 1), squares are constructed on each side of a right triangle and it is proved geometrically (not arithmetically) that the area of the square on the hypotenuse equals the sum of the areas on the other two sides.
- Historical context: Mathematicians debated the parallel postulate in *Elements* for over 2000 years, and the eventual disproving of this led to the development of other geometries (such as spherical and hyperbolic geometries).
- An English version of the *Elements* is easily accessible (both physically and digitally) in India. Though there have been many efforts to translate the *Elements* into Indian languages they are not readily accessible.

Anecdotes about *Elements*

- A KV Maths Teacher: “This is the first time I feel like I am using the geometry I learnt”¹⁰
- Narmada Devi (Journalist): Why were some mathematical techniques invented? And have significant works in Mathematics, such as *Principia Mathematica*, been translated into Tamil?⁹
- Ricci (Chinese Translator of *Elements* writing about it in his diary): “the Elements could function as a crash course in logic.”⁸

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Book 1 of Euclid’s *Elements* in Tamil:
<https://www.imsc.res.in/outreach/EE1861/>



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